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Adaptive Variable Structure Control of Aircraft with an Unknown High-Frequency Gain Matrix

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This paper presents nonlinear adaptive and sliding-mode flight control systems for the roll-coupled maneuvers of aircraft. It is assumed that the parameters of the aircraft, as well as its high-frequency gain matrix, are unknown. Based on a backstepping design approach, nonlinear control laws (an adaptive variable structure control law and an adaptive control law) for the trajectory control of the roll angle, angle of attack, and sideslip angle using the aileron, elevator, and rudder are derived. The decomposition of the high-frequency gain matrix is used for the derivation of singularity-free flight control laws. An additional advantage of the control laws lies in the choice of design parameters of matrix decomposition for shaping the response characteristics. In the closed-loop system, the roll angle, angle of attack, and sideslip angle trajectories asymptotically follow the reference output trajectories. Simulation results are presented that show that in the closed-loop system, simultaneous longitudinal and lateral maneuvers are precisely performed in spite of the uncertainties in the aircraft parameters using each control system.

Nomenclature

g	=	gravitational acceleration, $\mathrm{m/s^2}$
I_x, I_y, I_z	=	moments of inertia about principal axes, $\mathrm{kg \cdot m^2}$
i_1	=	nondimensional inertia coefficient, $(I_z - I_y)/I_x$
i_2	=	nondimensional inertia coefficient, $(I_z - I_x)/I_y$
i_3	=	nondimensional inertia coefficient, $(I_y - I_x)/I_x$
l	=	rolling moment per I_x, $1/\mathrm{s}^2$
m	=	pitching moment per I_y, $1/\mathrm{s}^2$
n	=	yawing moment per I_z, $1/\mathrm{s}^2$
V	=	velocity of the aircraft center of mass, km/s
y	=	side force over aircraft mass and speed, 1/s
z	=	aerodynamic force along the z axis (over mass and speed), 1/s
α, β	=	angle of attack and sideslip angle, rad
$\delta a, \delta r, \delta e$	=	aileron, rudder, and elevator deflection angles, rad
θ, ϕ	=	pitch angle and angle of bank, rad

I. Introduction

GEOMETRIC nonlinear control theory has provided powerful tools for the systematic design of feedback control laws for complex nonlinear systems. For trajectory control of nonlinear systems, the input–output and exact linearization techniques have been widely applied. The dynamic models of modern high-performance aircraft operating in a large flight envelope have significant nonlinearity. Apparently, flight controllers designed using linearized aircraft models cannot provide stability in the entire flight envelope.

The feedback linearization (dynamic input–output map inversion) technique has played an important role in the design of flight control systems for nonlinear maneuvers [1–8]. However, the application of this approach requires the complete knowledge of the system dynamics, because one must cancel the nonlinear functions appearing in the tracking error dynamics. But the assumption of complete knowledge of nonlinear aerodynamic characteristics of aircraft is unrealistic. Although attempts have been made to analyze the robustness of inverse controllers and robust inverse flight controllers have been developed [9–11], the analysis of robustness is based on μ synthesis using linearization.

Considerable research has been done for designing nonlinear flight controllers in the presence of parametric uncertainties. Variable structure control (VSC) (sliding-mode control) theory [12] has been applied for designing flight controllers for uncertain aircraft models [13–16]. The variable structure (VS) control law is a discontinuous function of the state variables and the bounds on the uncertain functions are used for its design. Unlike the VSC theory, nonlinear adaptive control methods do not require uncertainty bounds. Instead, an adaptive control system includes an adaptation mechanism for tuning the time-varying controller gains. In the past, a variety of adaptive flight controllers have been developed [17–27]. Because the aircraft models have mismatched uncertainties, backstepping design techniques [28] have been applied to derive stable adaptive control systems [17–19,23]. The backstepping design method is recursive in nature and the design is completed in several steps, which depends on the relative degree of the controlled output variables. For aircraft models with unknown functions, adaptive laws have been developed using neural networks for function approximation [18–26]. In this approach, unknown functions must be estimated for compensation. An adaptive-critic-based neural architecture has been presented in [27] for the design of an optimal flight controller. Recently, research has focused on adaptive design with state and control constraints [19,23].

A key technical issue in the design of adaptive control laws for multi-input, multi-output (MIMO) uncertain systems is the dynamic interaction between the system inputs and outputs. Unlike the single input systems, adaptive feedback linearizing control laws for MIMO systems require online tuning of the input matrix [16,17]. But the estimated matrix can become singular during parameter adaptation, causing unbounded control input. One can try to use the parameter projection method [29], but this method requires the knowledge of the domain of the parameter space in which the estimated input matrix is nonsingular. Of course, one can design VSC laws for uncertain nonlinear aircraft models, but it requires certain restriction on the uncertainties in the input matrix [30].

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To deal with large uncertainties in the input matrix of MIMO systems, adaptive control designs based on matrix decomposition [31] have been proposed [32,33]. For a hypersonic aircraft model, Xu et al. [25] factorized the input matrix into the product of a known regressor matrix and a diagonal matrix and developed an adaptation scheme for tuning the parameters of the diagonal matrix for control. For adaptive design, *LDU*, *LDS*, and *SDU* decompositions of the high-frequency gain matrix have been found to be useful. Readers may refer to Tao [33] for an adaptive backstepping design for MIMO systems based on *SDU* decomposition for linear systems. Of course, the backstepping design method is equally applicable to nonlinear systems. Recently, an adaptive law for a nonlinear aeroelastic system with two control surfaces using a matrix decomposition has been developed [34]. Adaptive laws developed based on matrix decomposition are singularity-free. As such, it is of interest to design flight control systems for uncertain aircraft models using matrix decomposition.

The contribution of this paper lies in the design of two nonlinear control laws (an adaptive variable structure control law and an adaptive control law) for the roll-coupled maneuvers of an aircraft model based on the decomposition of the input matrix. It is assumed that the aircraft parameters, including its input matrix, are unknown. For the purpose of control, the roll angle ϕ, angle of attack α, and sideslip angle β are chosen as controlled output variables; these are controlled using aileron δa, elevator δe, and rudder δr. For the derivation of control systems, the *SDU* decomposition of the 3×3 input matrix (which is the high-frequency gain matrix for the linearized system) is obtained [31–33]. In this factorized form, the input matrix is expressed as the product of a positive definite symmetric matrix, a diagonal matrix, and a unit upper triangular matrix. For the derivation, it is assumed that only the signs of the leading principal minors of the input matrix are known. Based on a backstepping design technique, two control laws are developed. First an adaptive VSC system is designed that uses adaptation in its first step of design, followed by a VSC law in the second step of the backstepping procedure. Unlike the VSC design without matrix decomposition, the *SDU* decomposition permits the derivation of the control law for arbitrary perturbations in the input matrix. Then a second control system is designed that uses adaptation in each step of derivation. Unlike the adaptive VSC law, the second flight controller (adaptive law) does not require the knowledge of certain bounds on the parameter uncertainties. In the closed-loop systems, it is shown that the output tracking error asymptotically tends to zero. Simulation results are presented that show that both the adaptive VSC law and the adaptive control law accomplish precise control of (ϕ, α, β) in spite of the uncertainties in the aerodynamic and inertia parameters of the aircraft.

II. Aircraft Model and Control Problem

The equations of motion considered here are in the principal axes. The complete set of equations taken from Hacker and Oprisiu [35] and Rhoads and Schuler [36] of a swept-wing fighter are given by (see these references for the notation and terminology)

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} l_\beta \beta + l_q q + l_r r + (l_{\beta\alpha}\beta + l_{r\alpha} r)\Delta\alpha + l_p p - i_1 qr \\ \bar{m}_\alpha \Delta\alpha + \bar{m}_q q + i_2 pr - m_{\dot{\alpha}} p\beta + m_{\dot{\alpha}}(g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ n_\beta \beta + n_r r + n_p p + n_{p\alpha} p \Delta\alpha - i_3 pq + n_q q \\ q - p\beta + z_\alpha \Delta\alpha + (g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ y_\beta \beta + p(\sin\alpha_0 + \Delta\alpha) - r\cos\alpha_0 + (g_0/V)\cos\theta\sin\phi \\ p + q\tan\theta\sin\phi + r\tan\theta cos\phi \\ q\cos\phi - r\sin\phi \end{pmatrix} + \begin{pmatrix} \tilde{l}_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \\ \tilde{n}_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & z_{\delta e} \\ y_{\delta a} & y_{\delta r} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta a \\ \delta r \\ \delta e \end{pmatrix} \tag{1}$$

where $\tilde{l}_{\delta a} = l_{\delta a} + l_{\alpha\delta a}\Delta\alpha$ and $\tilde{n}_{\delta a} = n_{\delta a} + n_{\alpha\delta a}\Delta\alpha$. The mathematical model of the airplane ignores speed changes and contains only rudimentary representation of the aerodynamic nonlinearities. It is further assumed that $u \cong V$ and the velocity components v and w are small. As such, one has $\beta \cong v/V$ and $\alpha \cong w/V$. The subscripts α, β, $\dot{\alpha}$, p, q, r, δa, δr, and δe denote the partial derivatives of the rolling, pitching, and yawing moments (l, m, and n) as well as of forces (y and z) with respect to respective quantity. For example $y_\beta = (\partial y/\partial\beta)$, $m_{\dot{\alpha}} = (\partial m/\partial\dot{\alpha})$, $n_{\alpha\delta a} = (\partial^2 n/\partial\alpha\partial\delta a)$, etc. These assumptions are made here for simplicity, but the design method could be applied to models with more complete aerodynamics, and throttle control could be implemented for speed control.

Defining $x_1 = (\phi, \alpha, \beta)$, $x_2 = (p, r, q)$, $x_3 = \theta$, and $x = (x_1^T, x_2^T, x_3) \in R^7$, the system equation (1) can be written as

$$\begin{gathered} \dot{x}_1 = f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2 + G_{1f}\delta \\ \dot{x}_2 = f_2(x)w + G_2(x)\delta \qquad \dot{x}_3 = f_3(x_1, x_2) \end{gathered} \tag{2}$$

where f_{10} and f_2 are nonlinear vector functions, f_{11} is a nonlinear 3×2 matrix, f_3 is a nonlinear function,

$$\lambda = (z_\alpha, y_\beta)^T \in R^2, \quad w_p = (l_\beta, l_q, l_r, l_{\beta\alpha}, l_{r\alpha}, l_p, -i_1)^T \in R^7$$
$$w_r = (n_\beta, n_r, n_p, n_{p\alpha}, n_q, -i_3) \in R^6, \quad w_q = (\bar{m}_\alpha, \bar{m}_q, -m_{\dot{\alpha}}, i_2) \in R^4$$
$$w = (w_p^T, w_r^T, w_q^T)^T \in R^{17}$$

are the vectors of parameters, and $\delta = (\delta a, \delta r, \delta e)^T \in R^3$ is the control input vector. The vector functions f_{10} and f_2 and the matrix f_{11} can be easily obtained from Eq. (1) by comparison. The matrices $G_1(x_1, x_3)$, G_{1f}, and $G_2(x)$ are

$$\begin{aligned} G_1(x_1, x_3) &= \begin{bmatrix} 1 & \tan\theta\cos\phi & \tan\theta\sin\phi \\ -\beta & 0 & 1 \\ \sin\alpha_0 + \Delta\alpha & -\cos\alpha_0 & 0 \end{bmatrix} \\ G_{1f} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & z_{\delta e} \\ y_{\delta a} & y_{\delta r} & 0 \end{bmatrix} \\ G_2(x) = G_2(\Delta\alpha) &= \begin{bmatrix} \tilde{l}_{\delta a} & l_{\delta r} & 0 \\ \tilde{n}_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \end{bmatrix} \end{aligned} \tag{3}$$

We assume that the parameter vectors λ and w and the input matrix G_2 are unknown. Note that at $\theta = \pm\pi/2$, $G_1(x_1, x_3)$ becomes unbounded; therefore, we shall be interested in a region $\Omega \subset R^7$ of the state space, where

$$\Omega = \left\{ x \in R^7 : x_3 \neq \pm\frac{\pi}{2} \right\}$$

This implies that θ must be bounded away from $\pi/2$ by a nonzero constant $C_\theta < (\pi/2)$ (i.e., $|\theta| < C_\theta$). Let the controlled output vector be

$$y = x_1 = (\phi, \alpha, \beta)^T \tag{4}$$

Suppose that it is desired to track a smooth reference trajectory $y_r = x_{1r} = [\phi_r(t), \alpha_r(t), \beta_r(t)]^T$. We are interested in designing control laws such that in the closed-loop system, the output vector $y(t)$ asymptotically tracks the reference trajectory $y_r(t)$ in spite of the uncertainties in the aircraft parameters.

III. Control Laws

For the purpose of control law design, a simplified aircraft model will be used. Although aileron, rudder, and elevator produce forces, their primary effectiveness is as moment-producing devices. As such, ignoring the term $G_{1f}\delta$ and, in addition, neglecting the $\Delta\alpha$ dependence of G_2 gives a simplified model:

$$\dot{x}_1 = f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2$$
$$\dot{x}_2 = f_2(x)w + B\delta \qquad \dot{x}_3 = f_3(x_1, x_2) \tag{5}$$

where $B = G_2(0)$ is a constant matrix:

$$B = \begin{bmatrix} l_{\delta a} & l_{\delta r} & 0 \\ n_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \end{bmatrix} \tag{6}$$

Of course, the complete model equation (1) will be used for the closed-loop simulation later.

The adaptive control laws of [16,17] use inversion of the estimate of the input matrix B, which can become singular during parameter adaptation, causing divergence in the control surface deflection. This is avoided here using the matrix decomposition of the input matrix B according to Morse [31] (also see Tao [33]). The design is based on a backstepping method following Kristic et al. [28] in which the *SDU* decomposition of B is used for completing the second step of the design [33].

A. *SDU* Decomposition

There are several kinds of decomposition of the matrix B that can be used for the adaptive design. For this, the following assumption is made.

Assumption 1: The leading principal minors Δ_i $(i = 1, 2, 3)$ of B are nonzero.

For the aircraft model, the leading principal minors are $\Delta_1 = l_{\delta a}$, $\Delta_2 = l_{\delta a}n_{\delta r} - n_{\delta a}l_{\delta r}$, and $\Delta_3 = \bar{m}_{\delta e}\Delta_2$, and these are indeed nonzero. Under Assumption 1, it is possible to obtain the decomposition of B. For obtaining the SD_sU_s decomposition of B, first one obtains the *LDU* decomposition. The unique *LDU* decomposition of B is given by

$$B = LD^*U \tag{7}$$

where L is a unit (i.e., with all diagonal elements being 1) lower triangular, U is a unit upper triangular matrix, and

$$D^* = \mathrm{diag}\{d_1^*, d_2^*, d_3^*\} = \mathrm{diag}\left\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_2}\right\} \tag{8}$$

Then a nonunique *SDU* decomposition of B is given by Tao [33]:

$$B = SD_sU_s \tag{9}$$

where S is a symmetric positive definite matrix, U_s is a unit upper triangular matrix, and

$$D_s = \mathrm{diag}\left\{\mathrm{sgn}(\Delta_1)\gamma_1, \mathrm{sgn}\left(\frac{\Delta_2}{\Delta_1}\right)\gamma_2, \mathrm{sgn}\left(\frac{\Delta_3}{\Delta_2}\right)\gamma_3\right\} \tag{10}$$

where $\gamma_i > 0$ $(i = 1, 2, 3)$ are arbitrary real numbers. In fact, for any set of chosen γ_i, one has

$$S = LD^*D_s^{-1}L^T, \qquad U_s = D_s^{-1}L^{-T}D_sU \tag{11}$$

Computing L and U using Eq. (7) then substituting these in Eq. (11) gives the matrices S and U_s of the form

$$S = \begin{bmatrix} \gamma_1^{-1}|b_{11}| & \gamma_1^{-1}\mathrm{sgn}(b_{11})b_{21} & 0 \\ \gamma_1^{-1}\mathrm{sgn}(b_{11})b_{21} & (b_{21}^2\gamma_1^{-1} + |\Delta_2|\gamma_2^{-1})|b_{11}|^{-1} & 0 \\ 0 & 0 & \gamma_3^{-1}|b_{33}| \end{bmatrix}$$
$$U_s = \begin{bmatrix} 1 & u_{s1} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where $u_{s1} = [b_{12} - \gamma_2\gamma_1^{-1}b_{21}\mathrm{sgn}(\Delta_2)]b_{11}^{-1}$ and b_{ij} is the element in the ith row and jth column of matrix B.

B. Adaptive Sliding-Mode Control

In this subsection, an adaptive sliding-mode control law is designed using a backstepping procedure. The design is completed in two steps. In the first step, an adaptive law is designed for the virtual control input, and this is followed by the sliding-mode control law derivation.

Step 1) Define new coordinates:

$$z_1 = (x_1 - x_{1r}) + K_0\int_0^t (x_1 - x_{1r}) = \tilde{x}_1 + K_0x_s \qquad z_2 = x_2 - x_{2d} \tag{13}$$

where x_s is the integral of the tracking error $\tilde{x}_1$ satisfying $\dot{x}_s = \tilde{x}_1$ and $K_0 = \mathrm{diag}(k_{01}, k_{02}, k_{03})$ with $k_{0i} > 0$. Then taking the derivative of z_1 and using Eq. (5) gives

$$\begin{aligned}\dot{z}_1 &= f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2 - \dot{x}_{1r} + K_0\tilde{x}_1 \\ &= f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)(z_2 + x_{2d}) \\ &\quad - \dot{x}_{1r} + K_0\tilde{x}_1\end{aligned} \tag{14}$$

Let $\hat{\lambda}$ be an estimate of λ. In view of Eq. (14), one chooses the stabilizing signal x_{2d} of the form

$$x_{2d} = G_1^{-1}(x_1, x_3)[-f_{10}(x_1, x_3) - f_{11}(x_1)\hat{\lambda} - C_1z_1 + \dot{x}_{1r} - K_0\tilde{x}_1] \tag{15}$$

where $C_1 > 0$. Substituting Eq. (15) in Eq. (14) gives

$$\dot{z}_1 = -C_1z_1 + f_{11}(x_1)\tilde{\lambda} + G_1(x_1, x_3)z_2 \tag{16}$$

where $\tilde{\lambda} = \lambda - \hat{\lambda}$ is the parameter error.

To examine the stability property of Eq. (16), consider a positive definite quadratic Lyapunov function:

$$V_1(z_1) = \tfrac{1}{2}[z_1^Tz_1 + \tilde{\lambda}^T\Gamma_1\tilde{\lambda}] \tag{17}$$

where Γ_1 is a positive definite symmetric matrix. Its derivative along the solution of Eq. (16) gives

$$\dot{V}_1 = -C_1\|z_1\|^2 + z_1^Tf_{11}(x_1)\tilde{\lambda} + z_1^TG_1(x_1, x_3)z_2 + \tilde{\lambda}^T\Gamma_1\dot{\tilde{\lambda}} \tag{18}$$

To eliminate the unknown function of Eq. (18), one chooses the adaptation law of the form

$$\dot{\tilde{\lambda}} = -\dot{\hat{\lambda}} = -\Gamma_1^{-1}f_{11}^T(x_1)z_1 \tag{19}$$

Substituting Eq. (19) in Eq. (18) gives

$$\dot{V}_1 = -C_1\|z_1\|^2 + z_1^TG_1(x_1, x_3)z_2 \tag{20}$$

It follows from Eq. (20) that z_1 converges to zero if z_2 is zero. However, z_2 cannot be taken to be zero, because x_2 is not a control input.

Step 2) In step 2, the control input δ is selected so that z_2 converges to zero. Differentiating z_2 and using Eq. (5) gives

$$\dot{z}_2 = f_2(x)w + B\delta - \dot{x}_{2d} = f_2(x)w + SD_sU_s\delta - \dot{x}_{2d} \tag{21}$$

where the derivative of x_{2d} is

$$\begin{aligned}\dot{x}_{2d} &= \frac{\partial x_{2d}}{\partial x_1}[f_{10}(x_1,x_3) + f_{11}(x_1)\lambda + G_1(x_1,x_3)x_2] \\ &+ \frac{\partial x_{2d}}{\partial x_3}f_3(x_1,x_2) + \frac{\partial x_{2d}}{\partial x_{1r}}\dot{x}_{1r} + \frac{\partial x_{2d}}{\partial \dot{x}_{1r}}\ddot{x}_{1r} - \frac{\partial x_{2d}}{\partial \hat{\lambda}}\Gamma_1^{-1}f_{11}^T(x_1)z_1 \\ &\triangleq h_{10}(x,\hat{\lambda},t) + h_{11}(x_1,x_3,\hat{\lambda},t)\lambda \end{aligned} \tag{22}$$

where the argument t denotes the dependence of these functions on y_r and its derivatives, h_{10} and h_{11} are known functions, and

$$h_{11}(x_1,x_3,\hat{\lambda},t) = \frac{\partial x_{2d}}{\partial x_1}f_{11}(x_1) \tag{23}$$

We point out that one can derive a VSC law by setting $B = B^* + \Delta B$, where B^* is the nominal value of B and ΔB represents the uncertainty in B following Slotine and Li [12] and Khalil [30], without matrix decomposition. But this method requires that $\|\Delta B B^{*-1}\| < 1$, which restricts the allowable uncertainty in the input matrix.

Now consider a modified Lyapunov function:

$$V_2(z_1,z_2,\tilde{\lambda}) = V_1 + \tfrac{1}{2}z_2^TS^{-1}z_2 \tag{24}$$

Differentiating V_2 along the solution of Eq. (21) gives

$$\begin{aligned}\dot{V}_2 &= -C_1\|z_1\|^2 + z_1^TG_1(x_1,x_3)z_2 \\ &+ z_2^TS^{-1}[f_2(x)w + SD_sU_s\delta - \dot{x}_{2d}] \\ &= -C_1\|z_1\|^2 + z_2^T[G_1^T(x_1,x_3)z_1 \\ &+ S^{-1}\{f_2(x)w - \dot{x}_{2d}\} + D_sU_s\delta]\end{aligned} \tag{25}$$

Because U_s is a unit upper triangular matrix, one has

$$U_s\delta = \delta + (U_s - I_{3\times3})\delta \tag{26}$$

where $I_{k\times k}$ denotes an identity matrix of dimension k:

$$(U_s - I_{3\times3})\delta = \begin{bmatrix} 0 & u_{s1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \delta = \begin{bmatrix} u_{s1}\delta r \\ 0 \\ 0 \end{bmatrix}$$

In view of Eq. (26), it is possible to define the control signal δ as a function of $(I_{3\times3} - U_s)\delta$. No static loops appear because δa would depend on δr, and the remaining control inputs δr and δe are explicitly solvable.

To this end, it will be convenient to express $S^{-1}\{f_2(x) - \dot{x}_{2d}\}$ in a linearly parametrized form. Using Eq. (22), one has

$$\begin{aligned}S^{-1}\{f_2(x)w - \dot{x}_{2d}\} &= S^{-1}\{f_2(x)w - h_{10}(x,\hat{\lambda},t) - h_{11}(x_1,x_3,\hat{\lambda},t)\lambda\} \\ &= \psi_a(x,\hat{\lambda},t)w_a\end{aligned} \tag{27}$$

where the regressor matrix $\psi_a(x,\hat{\lambda},t) \in R^{3\times m}$ is a known function, m is an appropriate integer, and $w_a \in R^m$ denotes the collection of all the unknown parameters. Note that w_a consists of the elements of S^{-1}, as well as the product of these elements, with the unknown parameter vectors λ and w.

Let $w_a = w_a^* + \Delta w_a$ and $u_{s1} = u_{s1}^* + \Delta u_{s1}$, where w_a^* and u_{s1}^* are the nominal parameters and Δw_a and Δu_{s1} denote the unknown portions of w_a and u_{s1}, respectively. Substituting Eq. (26) in Eq. (25) and using Eq. (27) gives

$$\begin{aligned}\dot{V}_2 &= -C_1\|z_1\|^2 + z_2^T\{G_1^T(x_1,x_3)z_1 + D_s\delta \\ &+ [\gamma_1\operatorname{sgn}(\Delta_1)u_{s1}\delta r, 0, 0]^T + \psi_a(x,\hat{\lambda},t)w_a\}\end{aligned} \tag{28}$$

In view of Eq. (28), the control law is chosen as

$$\begin{aligned}\delta &= D_s^{-1}\{-G_1^T(x_1,x_3)z_1 - C_2z_2 - \psi_a(x,\hat{\lambda},t)w_a^* \\ &- [\gamma_1\operatorname{sgn}(\Delta_1)u_{s1}^*\delta r, 0, 0]^T - K\operatorname{sgn}(z_2)\}\end{aligned} \tag{29}$$

where $z_2 = (z_{21}, z_{22}, z_{23})^T$, $\operatorname{sgn}(z_2) = [\operatorname{sgn}(z_{21}), \operatorname{sgn}(z_{22}), \operatorname{sgn}(z_{23})]^T$, $K = \operatorname{diag}(k_1,k_2,k_3)$, and $k_i > 0$. We notice from Eq. (29) that indeed δr and δe are only functions of x, $\hat{\lambda}$, and t, and δr can be substituted to obtain δa. Substituting Eq. (29) in Eq. (28) gives

$$\begin{aligned}\dot{V}_2 &= -C_1\|z_1\|^2 - C_2\|z_2\|^2 - z_2^TK\operatorname{sgn}(z_2) + z_2^T\psi_a(x,\hat{\lambda},t)\Delta w_a \\ &+ z_{21}\gamma_1\operatorname{sgn}(\Delta_1)\delta r\Delta u_{s1} \\ \dot{V}_2 &\le -C_1\|z_1\|^2 - C_2\|z_2\|^2 + \sum_{i=1}^{3}|z_{2i}|[-k_i + \|\psi_{ai}\|\|\Delta w_a\|] \\ &+ \gamma_1|z_{21}||\delta r||\Delta u_{s1}|\end{aligned} \tag{30}$$

where ψ_{ai} is the ith row of the matrix ψ_a.

In view of Eq. (30), the gains k_i are chosen such that

$$\begin{aligned}k_1 &\ge \|\psi_{a1}(x,\hat{\lambda},t)\|\Delta w_{\mathrm{am}} + \gamma_1|\delta r||\Delta u_{\mathrm{sm}}| + \mu_1 \\ k_i &\ge \|\psi_{ai}(x,\hat{\lambda},t)\|\Delta w_{\mathrm{am}} + \mu_i, \qquad i = 2,3\end{aligned} \tag{31}$$

where $\|\Delta w_a\| \le \Delta w_{\mathrm{am}}$, $|\Delta u_{s1}| \le \Delta u_{\mathrm{sm}}$, and $\mu_i > 0$ is a constant. Using Eq. (31) in (30) gives

$$\dot{V}_2 \le -C_1\|z_1\|^2 - C_2\|z_2\|^2 - \sum_{i=1}^{3}\mu_i|z_{2i}| \tag{32}$$

In view of Eq. (32), it is concluded that $z_i(t)$ converge to zero as $t \to \infty$. This implies that $x_1 = (\phi,\alpha,\beta)^T \to x_{1r} = (\phi_r,\alpha_r,\beta_r)^T$ as $t \to \infty$ and therefore output trajectory tracking is accomplished. This completes the adaptive sliding-mode controller design.

It may be pointed out that the controller designed here forms only the inner loop and one has to design an outer loop for performing specific maneuvers. The outer loop superimposed over the inner loop generates desirable command signals that are input to the inner loop, yielding stable closed-loop responses and trajectory control. Readers may refer to [16], in which both the outer and inner loops have been designed (without matrix decomposition) for the three-dimensional inertial path following with stable state variable responses. Moreover, it is pointed out that this paper presents a general approach for the flight control system design for MIMO aircraft models using matrix decomposition. This approach can be extended for the trajectory control of other sets of controlled output vectors [e.g., (velocity roll angle, α, β), (p, α, β), (ϕ, θ, β), (heading angle, flight path angle, β), etc.]. Of course, the closed-loop responses depend on the choice of the controlled output vector. Indeed, if it is required to follow the pitch angle trajectory command, then it is appropriate to choose $y = (\phi,\theta,\beta)$ for the control system design. Readers may refer to [17,37], in which an adaptive controller for the trajectory control of (ϕ,θ,β) has been designed without SDU decomposition.

It is pointed out that the control law is well-defined for arbitrary perturbations of the input matrix B due to the SDU decomposition of B. The control law equation (29) is a discontinuous function of z_2 and uses adaptation law equation (19) for tuning the gain $\hat{\lambda}(t)$. The derived control law is a sliding-mode control law for the tracking of x_{2d} by x_2, if the feedback term $-G_1^T(x_1,x_3)z_1$ in Eq. (29) is dropped. It is known that the discontinuity in the control law may cause control chattering. But this can be avoided by a continuous approximation of the sgn function.

C. Adaptive Control Law

For the design of the control law of the previous subsection, the sliding-mode control design method is used in step 2. This requires the information on the bounds on the uncertain functions for the computation of the gain matrix K. Now an adaptive law is designed, which does not require these uncertainty bounds.

The adaptive design is accomplished by modifying step 2; the derivation of step 1 is still valid. Consider now a Lyapunov function:

$$V_a(z_1, z_2, \tilde{\lambda}, \tilde{\omega}_a, \tilde{u}_{s1}) = V_2 + \tfrac{1}{2}[\tilde{\omega}_a^T \Gamma_a \tilde{\omega}_a + \mathrm{sgn}(\Delta_1)\tilde{u}_{s1}^2 \Gamma_s] \quad (33)$$

where Γ_a is a positive definite symmetric matrix, $\Gamma_s > 0$, $\tilde{w}_a = w_a - \hat{w}_a$, $\tilde{u}_{s1} = u_{s1} - \hat{u}_{s1}$, and $\hat{w}_a$ and $\hat{u}_{s1}$ are the estimates of w_a and u_{s1}, respectively. Using Eq. (28), its derivative can be written as

$$\begin{aligned}\dot{V}_a = {} & -C_1\|z_1\|^2 + z_2^T\{G_1^T(x_1, x_3)z_1 + D_s\delta \\ & + [\gamma_1 \mathrm{sgn}(\Delta_1)u_{s1}\delta r, 0, 0]^T + \psi_a(x, \hat{\lambda}, t)w_a\} \\ & + \tilde{\omega}_a^T \Gamma_a \dot{\tilde{\omega}}_a + \Gamma_s \tilde{u}_{s1}\dot{\tilde{u}}_{s1} \end{aligned} \quad (34)$$

In view of Eq. (34), the adaptive law is chosen as

$$\begin{aligned}\delta = {} & D_s^{-1}\{-G_1^T(x_1, x_3)z_1 - \psi_a(x, \hat{\lambda}, t)\hat{w}_a \\ & - C_2 z_2 - [\gamma_1 \mathrm{sgn}(\Delta_1)\hat{u}_{s1}\delta r, 0, 0]^T\}\end{aligned} \quad (35)$$

where $C_2 > 0$. Substituting Eq. (35) in Eq. (34) yields

$$\begin{aligned}\dot{V}_a = {} & -C_1\|z_1\|^2 - C_2\|z_2\|^2 + z_2^T\psi_a(x, \hat{\lambda}, t)\tilde{w}_a \\ & + z_{21}\gamma_1 \mathrm{sgn}(\Delta_1)\tilde{u}_{s1}\delta r + \tilde{\omega}_a^T \Gamma_a \dot{\tilde{\omega}}_a + \Gamma_s \tilde{u}_{s1}\dot{\tilde{u}}_{s1}\end{aligned} \quad (36)$$

Now the adaptation rule can be chosen as

$$\begin{aligned}\dot{\tilde{w}}_a &= -\dot{\hat{w}}_a = -\Gamma_a^{-1}\psi_a^T(x, \hat{\lambda}, t)z_2 \\ \dot{\tilde{u}}_{s1} &= -\dot{\hat{u}}_{s1} = -\Gamma_s^{-1}\mathrm{sgn}(\Delta_1)\gamma_1\delta r z_{21}\end{aligned} \quad (37)$$

to eliminate unknown functions in Eq. (36). Substituting Eq. (37) in Eq. (36) gives

$$\dot{V}_a \le -C_1\|z_1\|^2 - C_2\|z_2\|^2 \quad (38)$$

Because V_a is a positive definite function of z_1, z_2, $\tilde{\lambda}$, $\tilde{w}_a$, $\tilde{u}_{s1}$, and $\dot{V}_a \le 0$, it follows that z_1, z_2, $\tilde{\lambda}$, $\tilde{w}_a$, and $\tilde{u}_{s1} \in L^\infty[0, \infty)$ (the set of bounded functions). Integrating Eq. (38) gives

$$\int_0^\infty (C_1\|z_1(t)\|^2 + C_2\|z_2(t)\|^2)\,\mathrm{d}t \le V_a(0) - V_a(\infty) < \infty \quad (39)$$

which implies that $z_i \in L^2[0, \infty)$ (the set of square integrable functions). According to Eqs. (16) and (21), $\dot{z}_i \in L^\infty[0, \infty)$ for $x(t) \in \Omega$. Now invoking Barbalat's lemma [28,33], one concludes that $z_i(t) \to 0$ as $t \to \infty$. This implies that $(\phi, \alpha, \beta)^T \to (\phi_r, \alpha_r, \beta_r)^T$ as $t \to \infty$, and the output tracking error converges to zero. This completes the adaptive law derivation.

Here we have considered the control of (ϕ, α, β). However, it is pointed out that this design approach using matrix decomposition can be extended for the adaptive trajectory control of (ϕ, θ, β) or other sets of suitably chosen controlled output vectors for performing desirable maneuvers.

IV. Simulation Results

In this section, simulation results are presented for the aircraft model of [35,36] for flight condition 1 (FC 1) ($M = 9$ and $H = 20{,}000$ ft) and FC 2 ($M = 0.7$ and $H = 0$ ft) using the adaptive VSC law as well as the adaptive control law. The complete set of aerodynamic parameters is provided in [35,36]. Although the control laws were derived for the simplified model, the nonzero parameters $y_{\delta a}$ and $z_{\delta e}$ are retained to include the effect of control forces. Furthermore, $\Delta\alpha$ dependence of the input matrix $G_2(\Delta\alpha)$ is introduced for a realistic simulation, even though for the derivation of the control laws, $G_2(0)$ was used. Because the value of $y_{\delta r}$ is not given in [36], it is taken to be zero. Here, $\alpha_0 = 1.5$ deg and $\theta_0 = 0$. It is also assumed that the initial estimate $\hat{\lambda}(0) = 0$.

The reference roll angle trajectory $\phi_r(t)$ and reference angle of attack trajectory $\alpha_r(t)$ are generated by third-order filters of the form

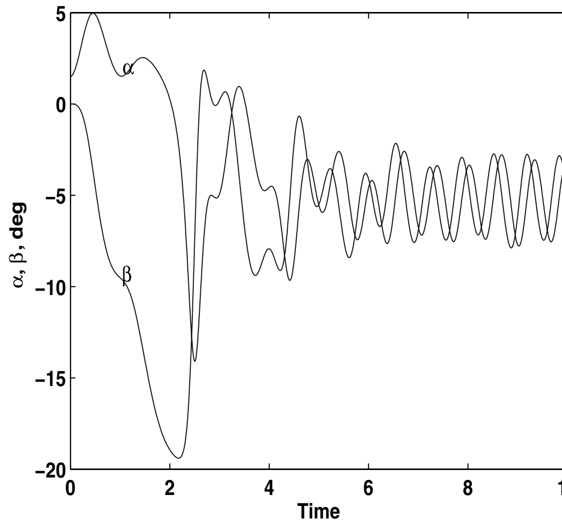

Fig. 1 Airplane response to combined aileron (20 deg) and elevator (−3.1 deg) inputs, showing the effect of coupling.

$$(s^3 + 3\lambda_r s^2 + 3\lambda_r^2 s + \lambda_r^3)\phi_r = \lambda_r^3\phi^*$$

and

$$(s^3 + 3\lambda_r s^2 + 3\lambda_r^2 s + \lambda_r^3)\alpha_r = \lambda_r^3\alpha^*$$

where ϕ^* and α^* are the target values for the roll angle and the angle of attack, respectively. The poles of the command generators are chosen to be at -1.5 by selecting $\lambda_r = 1.5$. The initial conditions for the command generators are $\alpha_r = 1.5$ deg and $\phi_r(0) = \dot{\phi}_r(0) = \ddot{\phi}_r(0) = \dot{\alpha}_r(0) = \ddot{\alpha}_r(0) = 0$. The reference sideslip angle $\beta_r(t)$ is equal to 0. The target roll angle and the angle of attack are $\phi^* = 360$ deg and $\alpha^* = 10$ deg, respectively. Thus, it is desired to roll the aircraft 360 deg and simultaneously change the angle of attack to 10 deg.

It is well known that implementation of the discontinuous VSC law can cause control chattering. For this reason, for simulation, the sgn function is replaced by a sat function given by

$$\mathrm{sat}(v) = \left\{\begin{array}{c} \frac{1}{\epsilon}v, |v| \le \epsilon \\ \mathrm{sgn}(v), |v| > \epsilon \end{array}\right\}$$

where $\epsilon = 0.9$ and $v \in R$.

First the uncontrolled aircraft model is simulated. The open-loop responses of the airframe with typical control deflections of $\delta a = 20$ deg and $\delta e = -3.1$ deg with $\delta r = 0$ are shown in Fig. 1. We observe that the motion of the variables are highly coupled and oscillatory. (Here only α and β plots are shown.) For a good airplane response, one would expect little sideslip. The irregular response, with rapid divergence of the sideslip angle to -20 deg, is typical of roll-coupled difficulties experienced with this example fighter aircraft [38].

A. Adaptive VSC

First the closed-loop system, including the complete aircraft model Eq. (1) and the adaptive VSC law Eq. (29), is simulated. For the adaptive VS control, the selected gains and matrices are $\Gamma_1 = 20I_{2\times2}$, $C_1 = 10$, $C_2 = 20$, $K = 2I_{3\times3}$, and $K_0 = I_{3\times3}$. Although one can use the inequalities equation (31) for the computation of the gains k_i, these inequalities provide only sufficient conditions for stability. Therefore, for simplicity, constant gains k_i, obtained by observing the simulation results, were used. The design parameters in the *SDU* decomposition are $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, and $\gamma_3 = 0.1$. For simulation, first the aircraft model at FC 1 is considered. For the computation of the control law, the selected nominal parameters are $w_a^* = 0 \in R^{32}$ and $u_{s1}^* = 0 \in R$. Therefore, one has $\Delta w_a = w_a$ and $\Delta u_{s1} = u_{s1}$, where w_a and u_{s1} are the actual parameters of the aircraft at FC 1. Thus, the uncertainty in the

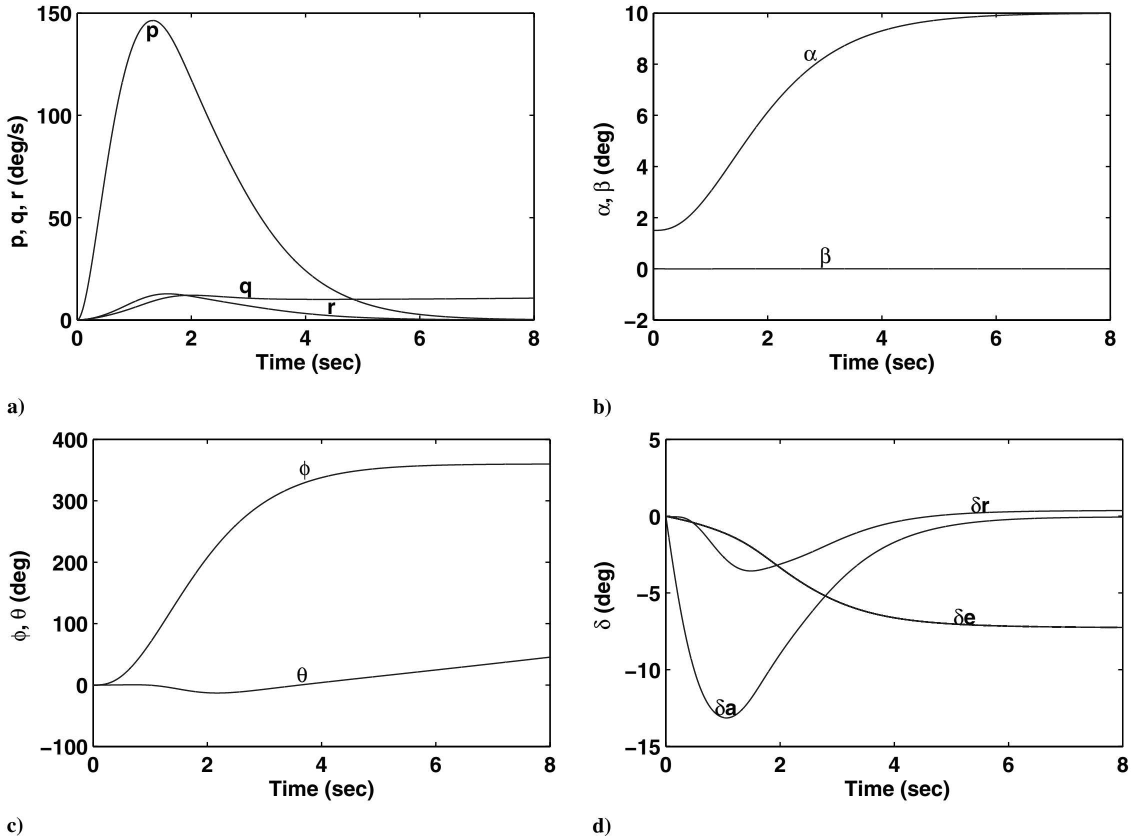

Fig. 2 Adaptive VS control of (ϕ, α, β) at FC 1 with 100% uncertainty in w_a and u_{s1}: a) angular velocities (p, q, r) deg /s; b) (α, β) deg; c) (ϕ, θ) deg; and d) control surface deflections $(\delta a, \delta r, \delta e)$ deg.

parameters is 100%. Of course, this is rather a worse choice of uncertainties, but it was made only to show the robustness of the control law. The selected responses are shown in Fig. 2. We observe trajectory control of the roll angle and the angle of attack to the target values within 5 s. The sideslip angle remains small (within 5×10^{-3} deg). It is found that the parameters γ_i used in the decomposition of the input matrix B provide flexibility in shaping the surface deflections. The control magnitude is on the order of $\delta = (-13.1, -3.6, -7.3)^T$ deg. The angular velocities are on the order of $(p, q, r) = (146.5, 12, 12.8)$ deg /s.

To examine the robustness of the adaptive VSC system, simulation is now performed for the aircraft model at FC 2. The feedback gains and initial values of the parameters $[C_1, C_2, K_0, K, \Gamma_1, \hat{\lambda}(0)]$ for Fig. 2 used at FC 1 are retained. Smooth responses are observed in this case too, as seen in Fig. 3. The control magnitude is on the order of $\delta = (-14.5, -2.5, -3.5)^T$ deg.

As mentioned earlier, the parameters γ_1, γ_2, and γ_3 of the *SDU* decomposition offer flexibility in the design. Based on the simulation results, it is observed that larger values of γ_1, γ_2, and γ_3 result in large sideslip (e.g., for $\gamma_1 = \gamma_2 = \gamma_3 = 50$, it is found that the peak value is about 1 deg, which is significantly larger than β of Fig. 2). However, there are no significant changes in other responses for the chosen design parameters.

B. Adaptive Control

Now, similar to adaptive VS control, simulations are performed for flight conditions 1 and 2 using the adaptive control law. The gains and matrices selected for adaptive control are $\Gamma_1 = 20I_{2\times 2}$, $\Gamma_a = 2I_{32\times 32}$, $\Gamma_s = 1$, $C_1 = 4$, and $C_2 = 4$. The design parameters chosen for *SDU* decomposition are $\gamma_1 = 1$, $\gamma_2 = 100$, and $\gamma_3 = 0.2$. The initial estimate $\hat{w}_a(0)$ of the parameter vector w_a and $\hat{u}_{s1}(0)$ are taken to be zero, where w_a and u_{s1} are the actual parameters of the aircraft at FC 1. The closed-loop system, including the control law Eq. (35) and adaptation laws Eq. (19) and (37), is simulated using the aircraft model Eq. (1) for FC 1. The selected responses are shown in Fig. 4. As in case A, trajectory control of the roll angle and the angle of attack to the target values is accomplished within 5 s, and the sideslip angle remains small (less than 0.08 deg). The control magnitude is on the order of $\delta = (-13.5, -3.7, -7.3)^T$ deg and the peak values of the angular velocities (p, q, r) are $(150, 12.2, 13)$ deg /s. Figure 5 shows the variation of some of the estimated parameters with time. Here, $\hat{w}_{a10}$ and $\hat{w}_{a13}$ are the estimates of $s_{I11}z_\alpha$ and $s_{I12}y_\beta$, respectively, where s_{Ijk} denotes the jkth element of S^{-1}.

To examine the performance of the adaptive control system at a different flight condition, simulation is performed using the aircraft model for FC 2, but the initial values of the parameters $\hat{w}_a(0)$, $\hat{u}_{s1}(0)$, and $\hat{\lambda}(0)$, as well as the feedback gains used for FC 1, are retained. Once again, smooth responses are observed, as seen in Fig. 6. The control magnitude is on the order of $\delta = (-14.6, -2.6, -3.5)^T$ deg. Figure 7 shows the variation of some of the estimated parameters with time.

Similar to the VS control, the design parameters in the *SDU* decomposition γ_1, γ_2, and γ_3 offer flexibility in the design. Based on the simulation results, it is observed that larger values of γ_i result in large sideslip angle (e.g., for $\gamma_1 = \gamma_2 = 100$ and $\gamma_3 = 1$, it is found that the peak value is about 0.7 deg). For large γ_3 (e.g., $\gamma_3 = 50$), α does not follow the desired trajectory very well. Small values of γ_2 and γ_3 yield excellent results, as seen in the figures presented. However, these results were obtained for specific values of the remaining design parameters mentioned earlier.

To this end, a comparison of the two control laws is appropriate. For adaptive law design, we need to update 32 parameters, but for the adaptive VS control system, only two parameters are adapted. Apparently, from the viewpoint of implementation, the adaptive VS controller is preferable. However, one must note that often the use of saturation function in place of the signum function in the adaptive VSC law may yield steady-state error. In addition, for the derivation of VSC law, one needs to know the bounds on the uncertain

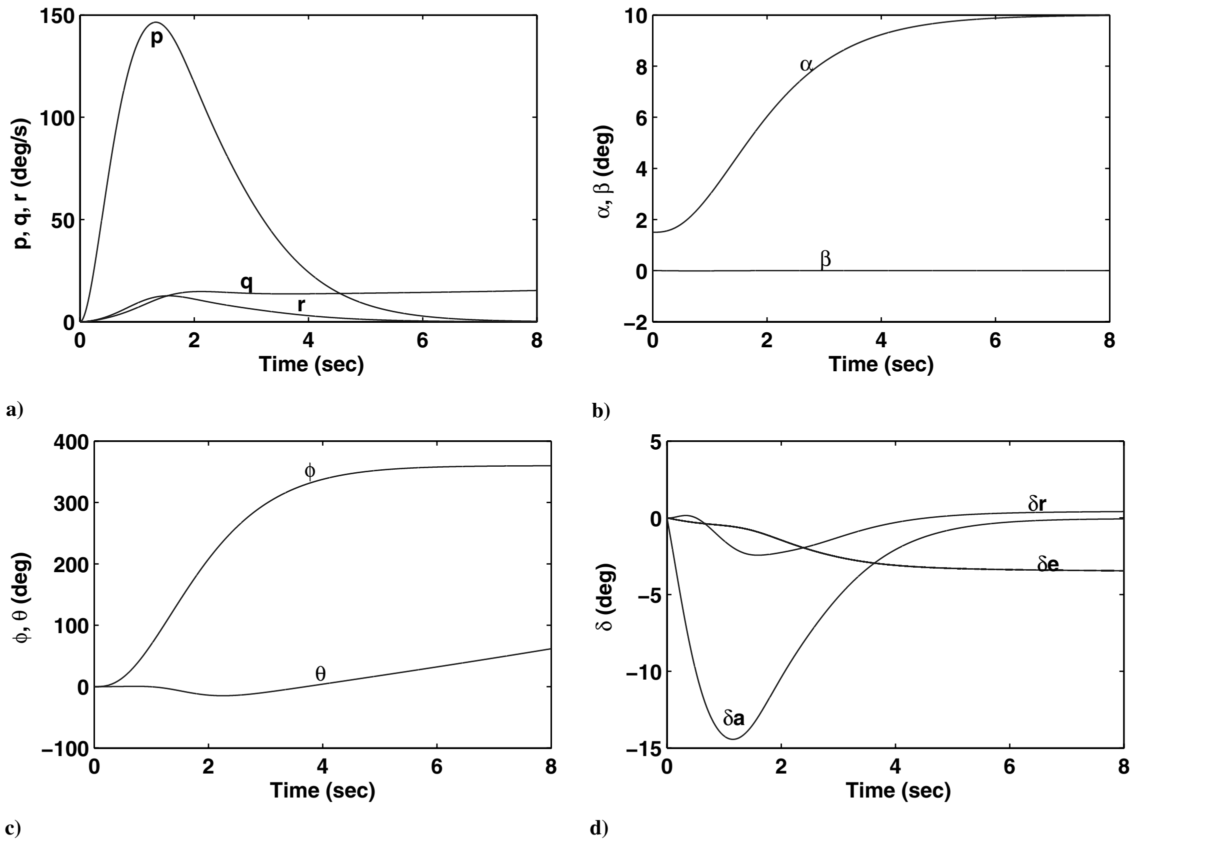

Fig. 3 Adaptive VS control of (ϕ, α, β) at FC 2 with the control law of FC 1: a) angular velocities (p, q, r) deg/s; b) (α, β) deg; c) (ϕ, θ) deg; and d) control surface deflections $(\delta a, \delta r, \delta e)$ deg.

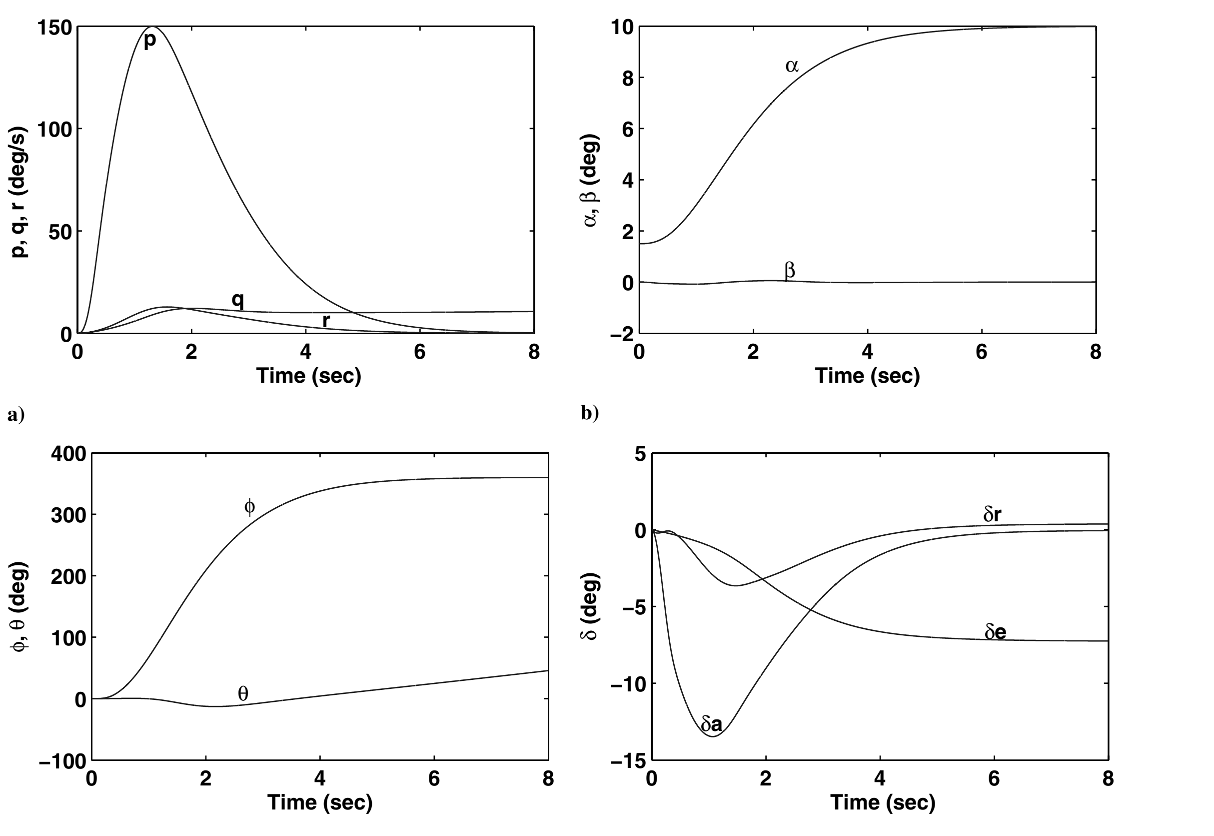

Fig. 4 Adaptive control of (ϕ, α, β) at FC 1 with 100% uncertainty in w_a and u_{s1}: a) angular velocities (p, q, r) deg/s; b) (α, β) deg; c) (ϕ, θ) deg; and d) control surface deflections $(\delta a, \delta r, \delta e)$ deg.

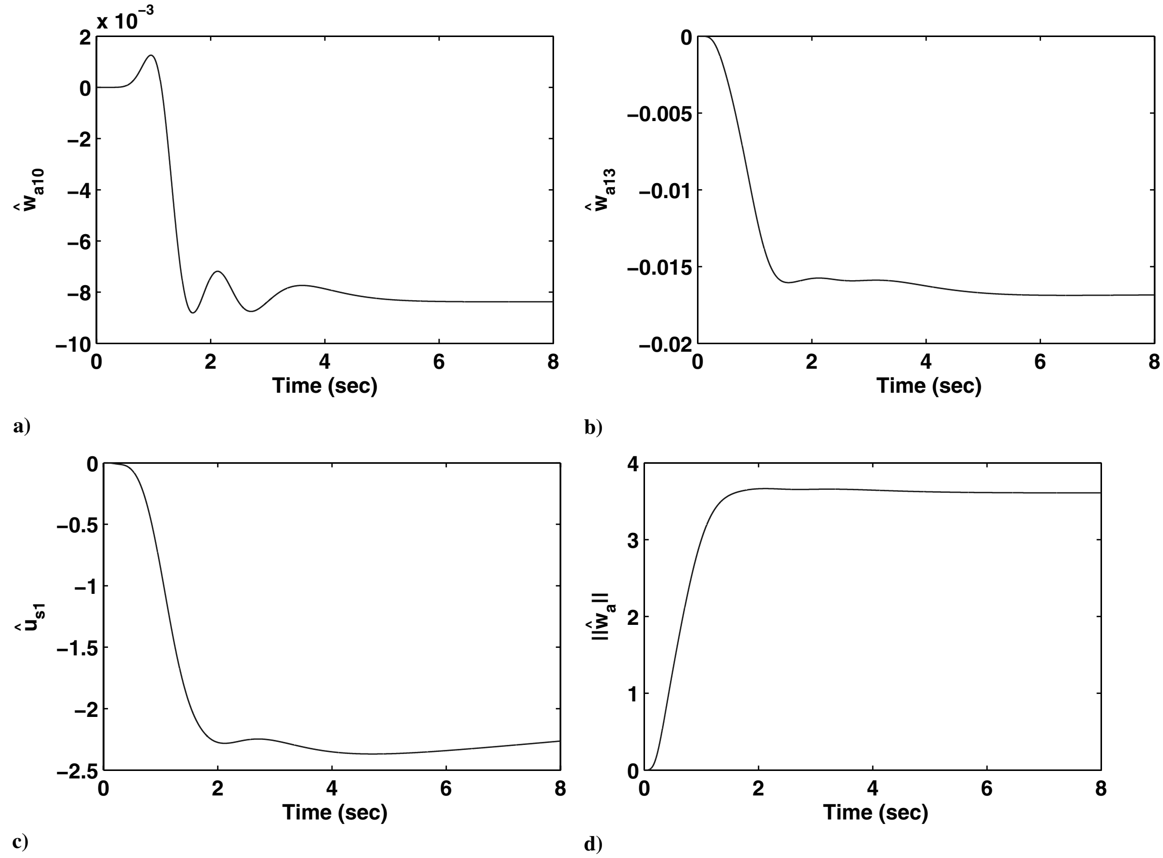

Fig. 5 Adaptive control of (ϕ, α, β) at FC 1 with 100% uncertainty in w_a and u_{s1}: a) variation in the estimated parameter $\hat{w}_{a10}$; b) variation in the estimated parameter $\hat{w}_{a13}$; c) variation in the estimated parameter $\hat{u}_{s1}$; d) variation in the estimated parameter vector norm $\|\hat{w}_a\|$.

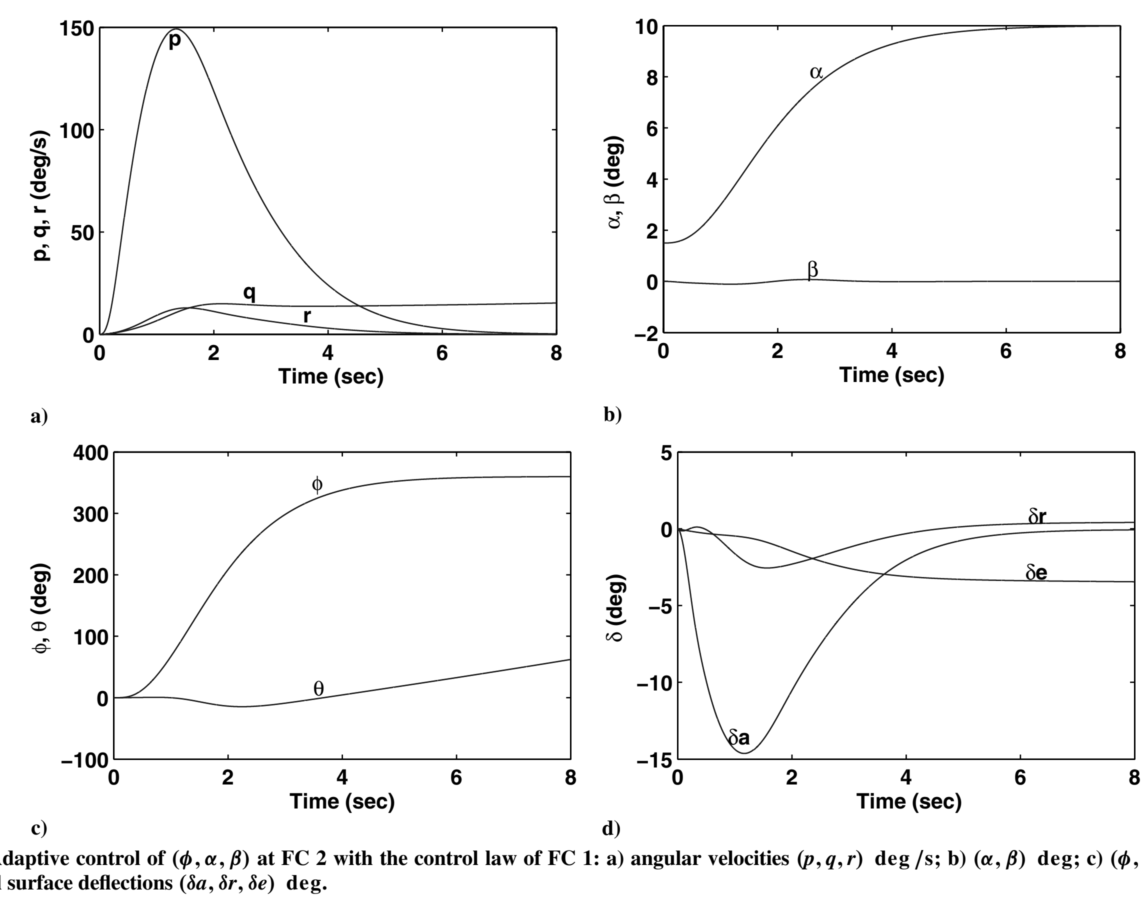

Fig. 6 Adaptive control of (ϕ, α, β) at FC 2 with the control law of FC 1: a) angular velocities (p, q, r) deg /s; b) (α, β) deg; c) (ϕ, θ) deg; and d) control surface deflections $(\delta a, \delta r, \delta e)$ deg.

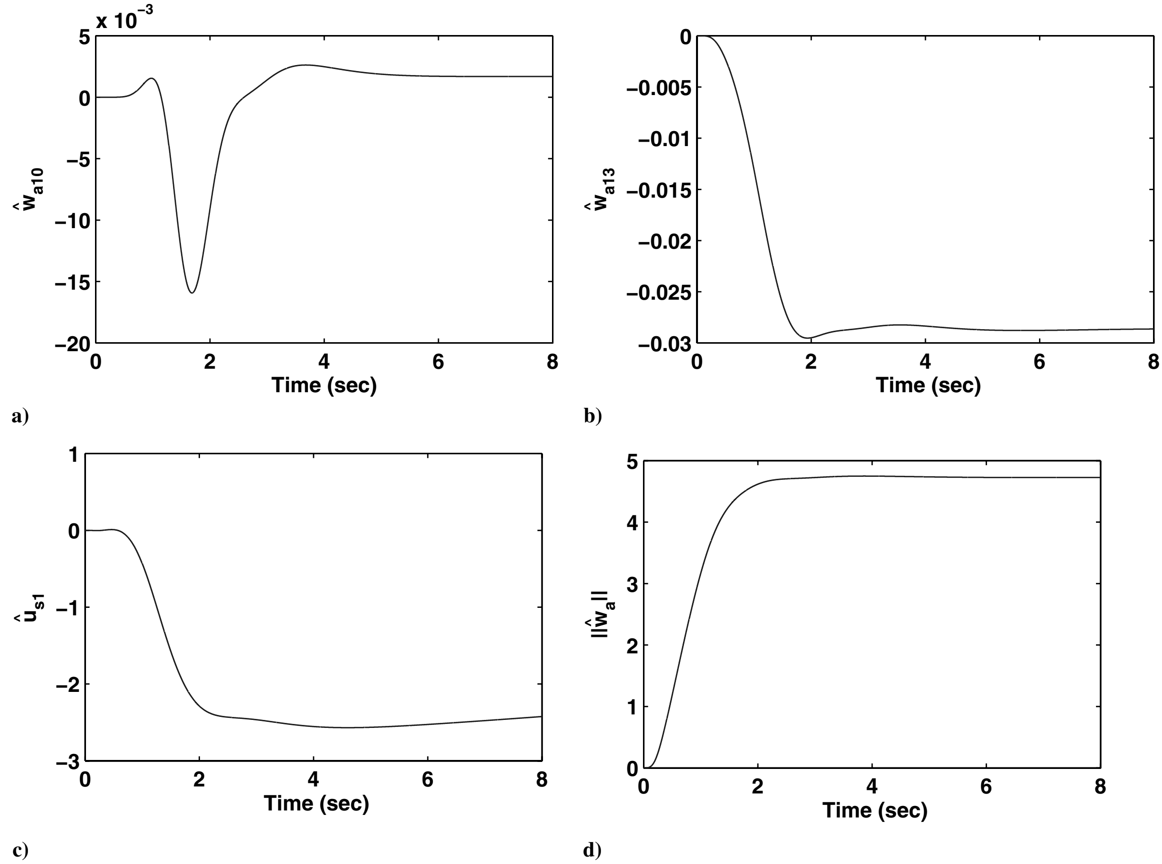

Fig. 7 Adaptive control of (ϕ, α, β) at FC 2 with the control law of FC 1: a) variation in the estimated parameter $\hat{w}_{a10}$; b) variation in the estimated parameter $\hat{w}_{a13}$; c) variation in the estimated parameter $\hat{u}_{s1}$; and d) variation in the estimated parameter vector norm $\|\hat{w}_a\|$.

parameters. Here, we have selected the controller gains after observing the simulated responses. Interestingly, for the aircraft model under consideration, the closed-loop system (including the adaptive VSC) and adaptive control laws with the selected design parameters have somewhat similar responses. Of course, for other choices of the controller gains, these two controllers will give different kinds of responses.

V. Conclusions

In this paper, the design of two nonlinear adaptive flight control systems for performing roll-coupled maneuvers was considered. It was assumed that the aircraft parameters and the high-frequency gain matrix were unknown. The adaptive variable structure control law and the adaptive control law were derived based on a backstepping design approach for tracking the roll angle, angle of attack, and sideslip angle trajectories using aileron, rudder, and elevator control surfaces. Matrix decomposition of the high-frequency gain matrix was used for the derivation of singularity-free control laws. Simulation results obtained showed that precise nonlinear roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of the uncertainty in the aerodynamic parameters using each of the derived control laws. There are several design parameters in both control laws that provide flexibility in shaping the response characteristics.

Appendix: Parameter Vectors w_a, w_a^*, u_{s1}, and u_{s1}^*

The actual parameter vector w_a for FC 1 is computed to be $w_a = [w_{a1} w_{a2} w_{a3}]^T$, where

$$w_{a1} = 1 \times 10^3[0.0022 \;\; -0.0000 \;\; -0.0218 \;\; 0.0002 \;\; 0.0003 \;\; -0.0086 \;\; -1.4934 \;\; 0.0183 \;\; 0.0000 \;\; -0.0029 \;\; -0.0004 \;\; 0.0000 \;\; 0.0000]$$

$$w_{a2} = 1 \times 10^3[-0.0000 \;\; 0.0002 \;\; 0.0009 \;\; 0.0000 \;\; -0.0000 \;\; 0.0000 \;\; 0.0022 \;\; -0.0000 \;\; 0.0000 \;\; 0.0000 \;\; 0.0000 \;\; -0.0002 \;\; -0.0000]$$

$$w_{a3} = 1 \times 10^3[0.0000 \;\; -0.0004 \;\; -0.0000 \;\; -0.0000 \;\; -0.0000 \;\; -0.0000]$$

and $u_{s1} = 0.1665$. The nominal parameters for design are $w_a^* = 0$ and $u_{s1}^* = 0$.

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